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# THE MODEL FOR OPTIMAL CHARGE IN THE BALL MILL

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**Abstract**

*Size of grinding media (balls) in ball mill strongly affect on specific mill capacity and grain size of mill product which further play great role in efficiency of applying concentration method of useful minerals, particularly in flotation.*

*Experimental results from dry grinding of quartz, as a monomineral material, were presented in this paper. Quartz samples were prepared in form of four particular size fractions such as: (-0.8+0.63) mm; (-0.63+0.5) mm; (-0.5+0.4) mm and (-0.4+0.315) mm. In the other hand the balls were prepared in form of four charges with uniform diameters for each charge, as like: 20mm, 15mm, 11mm and 6mm. The grinding was performed in mutual combination of each quartz size fraction with each uniform ball charge. Based on grinding kinetics from above combination of quartz size fractions to uniform ball diameter charge, the optimal ball diameter for certain quartz size fraction were obtained. Consequently to these results a model for optimal ball charge with all diameters was defined and verifed.*

*The experiments of quartz grinding were carried out in the laboratory steel mill with steel balls in dry environs. Degree of mill feed was maintained on 40% of mill volume in all experiment as well as mill revolution was constant at value 85% of mill critical speed.*

**Key words:** *Grinding, grain size, ball mill.*

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## 1. Introduction

In recent time, special attention has been paid to the process of mineral raw materials' grinding from the point of view of mill capacity as well as from size distribution of grinding product and its influence on technological parameters of mineral raw materials' concentration (recovery and quality of concentrate).

Size distribution of balls in ball mill has a significant impact on decrease of normative consumption as well as on quality of size distributions of grinding product, mill capacity and specific energy consumption.

The basic condition that the ball for comminution of a particle of raw material must satisfy is that stress occuring in it on impact should exceed the tensile strength of mineral raw material particle. That is why the maximal ball diameter in the mill is determined depending on raw material size distribution. After establishment of maximal ball diameter it is necessary to form the charge of grinding media of different diameters. Size of the charge must be optimal for grinding the particular raw material.

Some of the results reported so far show that the best grinding effects would be achieved by charge of balls having size distribution similar to that of the feed at ball mill entry.

## 2. Experimental investigations

In this paper investigations are divided into two parts. In the first part of investigations a model has been defined for selection of optimal composition of ball charge according to size distribution as a function of size distribution of raw material being ground and the second part of investigation has been dedicated to testing of the defined method for determination of optimal ball size distribution as a function of size distribution of mono-mineral raw material (quartz).

Investigations have been performed in laboratory ball mill with the following characteristics: Mill diameter D = 158 mm; Mill length L = 198 mm; Relative mill revolution rate $\Psi = 0.85 n_k$; Ball mill filling coefficient $\varphi = 0.40$; Internal surface of ball mill shell - ribbed type; Grinding manner - dry.

For the requirements of the first part of investigations samples of quartz sand have been used of high purity(> 99 % $SiO_2$), separated into four narrow size distributions:(-0.80+0.63mm; -0.63+0.50mm; -0.50+0.40mm; -0.40+0.315mm) and four different ball charge where each particular charge has comprised balls of the same diameters. Characteristics of these charges are presented in table 1.

**Table 1.** Characteristics of ball charge

| Ball size (mm) | Charge mass $M_{ch}$ (g) | No.of balls n (items) | Ball mass (g) |
|---|---|---|---|
| 6 | 7171 | 8149 | 0.88 |
| 11 | 6920 | 1277 | 5.42 |
| 15 | 6729 | 482 | 13.97 |
| 20 | 6475 | 199 | 32.57 |

Sample mass has been selected such that the sample in loose state fills up total intermediary space between balls in the charge. Masses of quartz sand samples for requirements of the first part of investigations are presented in table 2.

**Table 2.** Quartz sand sample masses for the first part of investigations

| Ball size $d_b$ (mm) | Mass of sample for grinding (g) | | | |
|---|---|---|---|---|
| | -0,80+0,63 mm | -0,63+0,50 mm | -0,50+0,40 mm | -0,40+0,315 mm |
| 6 | 1080 | 1073 | 1100 | 1072 |
| 11 | 1116 | 1139 | 1148 | 1140 |
| 15 | 1136 | 1140 | 1136 | 1153 |
| 20 | 1155 | 1142 | 1124 | 1168 |

## 3. Results and discussion

Efficiency of grinding of narrow size distributions of quartz with ball charge with different size distributions has been estimated by grinding rate constant ***k*** in the first order grinding kinetics[5] and by mill specific capacity per ground product, reduced to the unit mass of ball charge $Q_S$.

In all experiments it has been established that grinding kinetics of narrow size distribution of quartz in laboratory ball mill proceeds according to the first order kinetics law[5]:

$$\frac{dR}{dt} = -kR \qquad (1)$$

Results of investigations clearly show that grinding rate of quartz of corresponding size depends on the size of balls used for its grinding. For all conducted grinding tests grinding rate constants ***k*** has been determined and they are presented in table 3.

**Table 3.** Grinding rate constants

| Ball size $d_b$ (mm) | Grinding rate constant, ***k*** | | | |
|---|---|---|---|---|
| | - 0,80+0,63 mm | - 0,63+0,50 mm | - 0,50+0,40 mm | - 0,40+0,315 mm |
| 6 | 0,0351 | 0,0872 | 0,1134 | 0,0959 |
| 11 | 0,0526 | 0,0861 | 0,0893 | 0,0646 |
| 15 | 0,0538 | 0,0752 | 0,0696 | 0,0496 |
| 20 | 0,0635 | 0,0619 | 0,0550 | 0,0336 |

Fig. 1. shows relationship between grinding rate constant k and ball size in charge $d_b$.

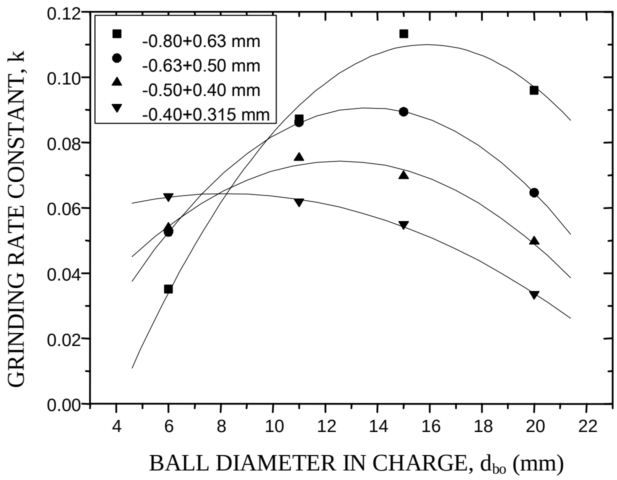


**Fig. 1.** Relationship of grinding rate constant ***k*** and ball size incharge in grinding of different narrow size distributions of quartz

It is obvious from the fig.1 that grinding rate constant for particular narrow size distributions of quartz depend to a large extent on the ball size in charge.

For each narrow size distribution of quartz there exists a ball diameter which gives the highest grinding rate, and that is, in fact, the optimal diameter of ball for the given quartz size.

Since ball charge of different diameters differ in mass, and also masses of samples of narrow size distribution of quartz are different, grinding efficiency , observed from the point of view of ball mill specific capacity per finished grinding product, reduced to the unit mass of ball charge is:

$$Q_S = \frac{Q}{M} = \frac{60 \cdot m_k \cdot \alpha}{t}, (\mathrm{kg/h \cdot kg}) \quad (2)$$

where: $Q_s$ - ball mill specific capacity per finished product, reduced to the unit mass of ball charge, (kg/h·kg)

Q - mill capacity per finished product, (kg/h)

M - charge mass, (kg)

m - mass of quartz sample for grinding, (kg)

$\alpha$ - contribution of ground narrow size distribution of quartz after grinding time t, (fractions of unity),

t - grinding time (min)

Ball mill specific capacity per finished product, reduced to unit mass of ball charge has been calculated for grinding time t=4 min and graphically presented in fig 2 depending on ball size in the charge.

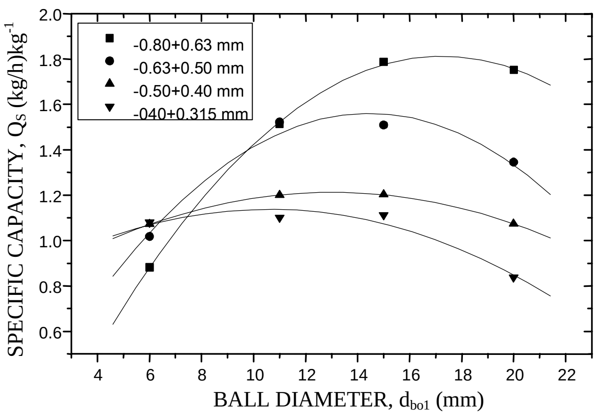


**Fig. 2.** Relationship of ball mill specific capacity per unit mass of ball charge and ball diameter in the charge

By graphical interpolation from fig.1 and 2, diameter of the balls has been determined which gives the highest grinding efficiency for a particular narrow size distribution of quarttz. These results are presented in table 4.

**Table 4.** Optimal ball diameter for grinding of narrow size distribution of quartz

| Quartz size distribution d (mm) | Mean diameter of narrow size distribution $d_s$ (mm) | Optimal ball diameter in charge $d_b$ (mm) | | |
|---|---|---|---|---|
| | | Per grinding rate constant ***k***, (fig. 5) | Per ball mill. spec.capacity per ball charge size unit $Q_S$, (fig. 6) | Mean value for optimal ball size |
| - 0,80 + 0,63 | 0,715 | 16,00 | 16,50 | 16,25 |
| - 0,63 + 0,50 | 0,565 | 13,50 | 14,00 | 13,75 |
| - 0,50 + 0,40 | 0,450 | 12,50 | 13,00 | 12,75 |
| - 0,40 + 0,315 | 0,357 | 8,00 | 9,00 | 8,50 |

It may be seen from table 4 that for both parameters for estimation of grinding efficiency very close values are obtained for the optimal ball diameter in charge.

Fig. 3 shows the relationship of the optimal ball diameter in charge and the mean diameter of the quartz grain size.

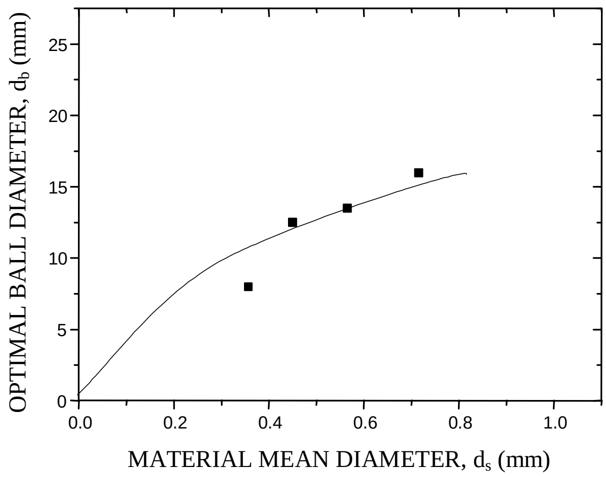


**Fig. 3.** Relationship of the optimal ball diameter in charge and the mean diameter of quartz grain

Graphical representation in fig. 3 shows the correlation relationship of the form:

$$d_{bo} = K d_m^n, \quad (mm) \tag{3}$$

where: $d_{bo}$ - optimal ball size, (mm)

$d_m$ - mean diameter of narrow size distribution of the material, (mm)

K and n - constants that depend on the type of raw material and which are determined experimentally.

Correlation relationship between ball diameter and size od quartz sand grains for the conditions in which these investigations have been carried out are defined by the following equation:

$$d_{bo} = 22{,}67 d_m^{0{,}87}, \quad (mm) \tag{4}$$

Correlation degree is very high and amounts to r= 0.95.

The equation (4) confirms a direct proportionality between the ball diameter and mean diameter of quartz grain, which is in accordance with theoretical expressions for the optimal ball diameter.[1,2,3,6]

For tests in the scope of this paper, we have adopted the mathematical model of Magdalinovic, which is presented here in details for better insight:

Size distribution of material which is ground in ball mill, in majority of cases, can be described by Gaudin-Schumann's equation:

$$D = 100\left(\frac{d}{k}\right)^n \tag{5}$$

where: D - cumulative undersize of the screen of mesh size (d), %

d - screen mesh size

k - theoretical maximal size of the material

n - exponent which depends on material size distribution

The optimal size distribution of ball mill charge should be defined by the equation:[1]

$$Y = 100\left(\frac{d}{d_{b_{\max}}}\right)^m, [d_{b_{\min}} \le d \le d_{b_{\max}}] \tag{6}$$

where: Y - cumulative contribution of ball of smaller diameter d, %

d - ball diameter

$d_{bmax}$- diameter of the largest ball, according to equation (8) m - exponent which depends on ball size distribution in charge and which should be equal to the exponent n in equation (5) ($m \cong n$)

$d_{bmin}$- minimal ball diameter which can operate efficiently

According to Magdalinovic's principle, exponents *n* and *m* in equations (5) and (6) should be equal: $m \cong n$. In other words,the optimal size distribution of ball mill charge is determined in the following way:

1. For the known size distribution of the material which is ground in ball mill, the value of exponent *n* is determined in Gaudin-Schumann's equation (5).
2. Maximal ball diameter $d_{bmax}$ in charge is determined, in accordance with grinding conditions and characteristic dimensions of the largest grains in the material, according to one of the known and given formulae in this paper.
3. Contribution of balls of different balls in charge,in the range from $d_{bmax}$ to $d_{bmin}$ is calculated according to the formula (6) in which m=n.

Results of investigations in the first part of this paper clearly show that the optimal composition of ball charge in the mill is conditioned by the size distribution of the material which is ground.

In order to confirm that the define model for establishment of the optimal composition of ball charge according to the size for grinding of corresponding size distribution of the material and given conditions is satisfactory, its testing has been performed.

For testing of the model for optimal size distribution of ball charge samples of quartz and copper ore have been formed of the size -0.8+0.315. Size distribution of samples is presented in table 5, graphically in fig.4 and by Gaudin-Schumann's equation (7).

**Table 5.** Size distribution of quartz sample

| Size distribution d (mm) | W (%) | D (%) | R (%) |
|---|---|---|---|
| -0,80+0,63 | 38 | 100 | 38 |
| -0,63+0,50 | 23 | 62 | 61 |
| -0,50+0,40 | 14 | 39 | 75 |
| -0,40+0,315 | 25 | 25 | 100 |

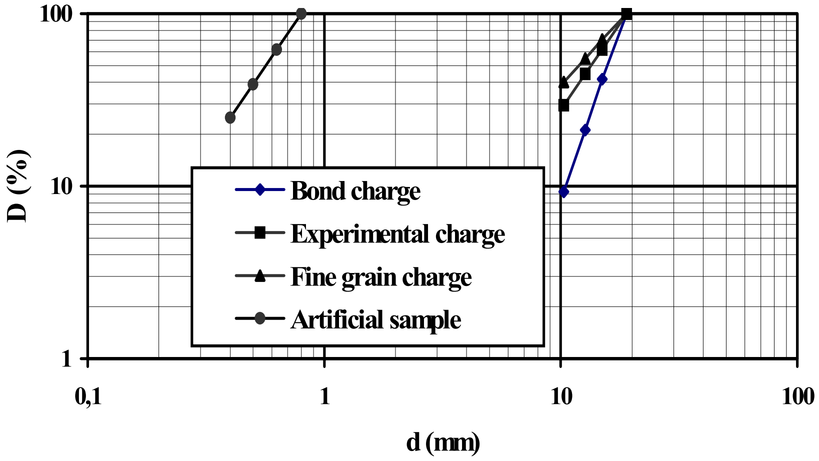


**Fig. 4.** Size distribution of the sample and size distribution of ball charge A, B and C

For the investigation the following three ball charges have been formed and used:

- Charge A ( Bond charge)
- Charge B (Experimental charge) and
- Charge C (Small size charge)

Charge B (Experimental charge) has been formed according to Magdalinovic's model in the following manner:

1. Sample size distribution, which is ground in ball mill, can be represented by Gaudin-Schumann's equation of the form:

$$D = 100\left(\frac{d}{0{,}8}\right)^2 \qquad (7)$$

2. Maximal ball diameter $d_{bmax}$ in charge, in accordance with grinding conditions and characteristic dimension of the largest grain size of the material, determined according to the formula (4):

$$d_{b\max} = 22{,}67(0{,}8)^{0{,}87} = 18{,}7 \quad (mm)$$

Based on these results, adopted maximal ball diameter is $d_{bmax}$ = 19mm.

3. Contribution of balls of different diameters in charge, in the range from $d_{bmax}$ to $d_{bmin}$ is calculated according to formula (8) where m=n=2:

$$Y = 100\left(\frac{d}{19}\right)^2, [10,3 \le d \le 19] \qquad (8)$$

$d_{bmin}$ is calculated according to the same formula as $d_{bmax}$ and the following value has been obtained: $d_{bmin}$ = 8.3 mm.

Based on these results and due to availability of that diameter of balls in laboratory, adopted minimal ball diameter is $d_{bmin}$ = 10.3 mm.

The charge has been formed from ball diameters which were available including 19; 15; 12.7 and 10.3 mm.

- Charge A (Bond charge) has been formed according to Bond hypothesis about the ball wear, when the charge is replenished only with balls of the largest diameter. Such charge is present in industrial ball mills. In this charge large size balls are predominant.
- Charge C (Small size), is formed from balls of smaller diameter.In this charge smaller balls are predominant.

Characteristics of charge size are presented in table 6 and graphical representation of characteristics of ball charge size are shown in fig.4.

**Table 6.** Characteristics of ball charge size

| Ball diameter (mm) | Ball mass (g) | Bond charge | | | | Experimental charge | | | | Small size charge | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | W (%) | D (%) | M (g) | No. balls | W (%) | D (%) | M (g) | No. balls | W (%) | D (%) | M (g) | No. balls |
| 19 | 27,2 | 58,3 | 100 | 4157 | 153 | 38 | 100 | 2732 | 100 | 29 | 100 | 2090 | 77 |
| 15 | 13,9 | 20,5 | 41,7 | 1480 | 106 | 23 | 62 | 1653 | 119 | 16 | 71 | 1159 | 84 |
| 12,7 | 8,3 | 11,9 | 21,2 | 853 | 103 | 14 | 39 | 1006 | 121 | 15 | 55 | 1089 | 131 |
| 10,3 | 4,5 | 9,3 | 9,3 | 667 | 149 | 25 | 25 | 1797 | 399 | 40 | 40 | 2898 | 648 |
| | | 100 | | 7147 | | 100 | | 7188 | | 100 | | 7236 | |

Sample mass has been selected in such manner that in loose state it fills up all intermediary area between balls in charge. Masses of quartz and copper ore samples are given in table 7.

**Table 7.** Masses of quartz samples

| Ball charge | Masses of quartz samples (g) |
|---|---|
| Bond charge | 914 |
| Experimental charge | 915 |
| Small size charge | 908 |

Grinding efficiency has been estimated by grinding rate constant *k* in the equation of the grinding kinetics of the first order and by specific mill capacity per finished product, reduced to charge mass unit.

Grinding rate constants $k_i$ in grinding of quartz sample with different ball charges have been calculated by the least square method and they are presented in table 8. Fig. 5 shows the graphical representation of the change of grinding rate constant for quartz sample with different ball charges.

**Table 8.** Grinding rate constants for artificial quartz sample by defined ball charges

| Test screen mesh size $d_t$ (mm) | Mean diameter of ball charge | | |
|---|---|---|---|
| | Small size charge $d_m$=13,1 mm | Experimental charge $d_m$=14,2 mm | Bond charge $d_m$=16,0 mm |
| 0,63 | 0,2980 | 0,3817 | 0,3691 |
| 0,5 | 0,2439 | 0,2696 | 0,2577 |
| 0,4 | 0,1552 | 0,1590 | 0,1433 |
| 0,315 | 0,1055 | 0,1188 | 0,1082 |
| 0,2 | 0,0426 | 0,0450 | 0,0429 |
| 0,09 | 0,0129 | 0,0131 | 0,0127 |

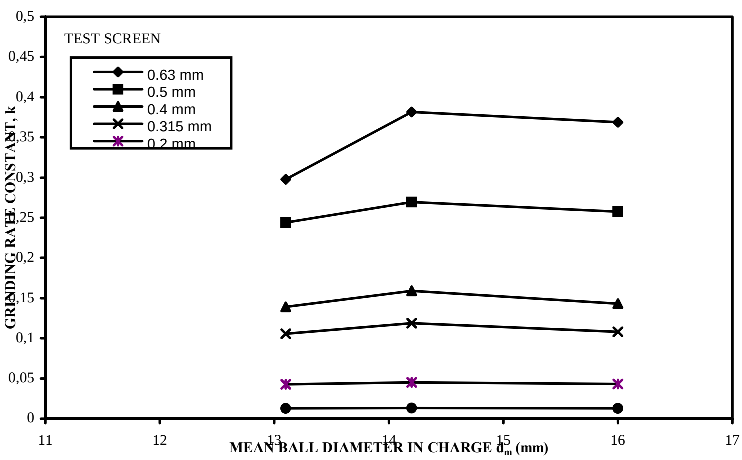


**Fig. 5.** Change of grinding rate constant of artificial quartz sample as a function of ball size

Values for specific mill capacity per finished product, reduced to unit mass of charge for quartz samples are presented in table 9, and fig. 6 shows graphical representation of dependence of specific capacity on ball diameter.

**Table 9.** Specific mill capacity per finished product, reduced to charge unit mass for quartz samples

| Test screen mesh size $d_t$ (mm) | Mean diameter of ball charge | | |
|---|---|---|---|
| | Small size charge $d_m$=13,1 mm | Experimental charge $d_m$=14,2 mm | Bond charge $d_m$=16,0 mm |
| 0,63 | 2,3501 | 2,4217 | 2,4168 |
| 0,5 | 2,0564 | 2,1523 | 2,1099 |
| 0,4 | 1,3771 | 1,4361 | 1,4249 |
| 0,315 | 0,7958 | 0,8901 | 0,8208 |
| 0,2 | 0,2939 | 0,3236 | 0,2893 |
| 0,09 | 0,0873 | 0,1054 | 0,0831 |

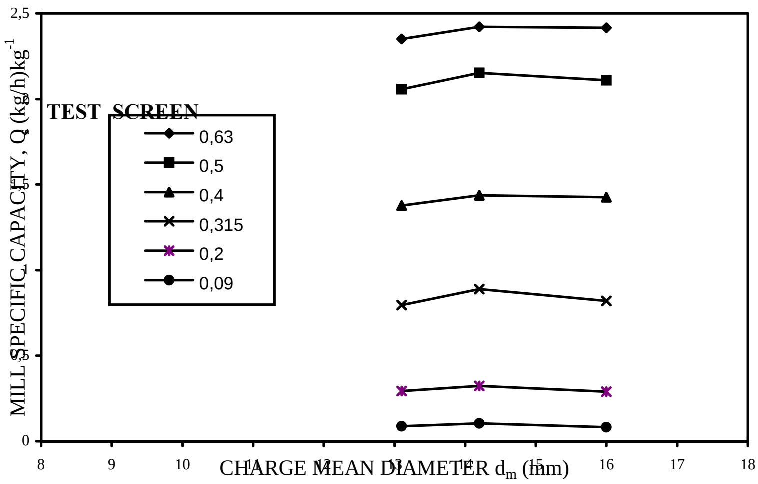


**Fig. 6.** Dependence of mill specific capacity on mean charge diameter

By observing the plots shown in fig. 5 and 6, one can clearly conclude that the value for grinding rate constant and mill specific capacity for all test screen mesh sizes is the largest in tests with the charge which has been formed according to Magdalinovic's model, i.e. experimental charge, which fully confirms that the correlation relationship shown by equation (4), is absolutely true.

## 4. Conclusion

Investigations in this paper are divided into two parts.

1. In the first part of investigations, dry-type grinding of narrow size distribution of quartz with ball charge of different diameters in laboratory ball mill has shown that between the optimal ball diameter ($d_{bo}$) and quartz grain size (d) there exists a direct correlation relationship of the form:
   $d_{bo} = 22{,}67 d_m^{0{,}87}, \quad (mm)$
2. In the second part of investigations testing of defined model has been carried out which has shown that the most efficient grinding is achieved by using the charge which is formed according to the previous model. i.e. experimental charge.

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