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# MATEMATICAL MODELS OF ACCIDENT SIMULATION AT THE FLOTATION TAILING DUMP "VALJA FUNDATA"

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**Abstract**

*Damages (accidents) at the flotation tailing dumps could cause at one area an ecological disaster of small or great sizes. Due to this, the investigations of such appearances are carried out for the aim of defining the executive analytical-mathematical models for estimation the dangers of possible accidents. Mathematical models for simulation of accident flow at flotation tailing dump, given in this work, are defined based on an analysis of an actual accident flow at "Valja Fundata" flotation tailing dump in Majdanpek (Serbia) in 1974. The evaluation of the eventual accident at any flotation tailing dump based on mathematical models given in this paperwork, could be done for any case where the height of flow over the abyss is sless than 117 m, while the average cross cut surface of the damage hole is up to 2.2 m$^2$.*

**Key words:** *Flotation tailing dumps, accident, mathematical models, depression funnel, flood wave.*

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## 1. Introduction

The flotation plant of the cooper mine Majdanpek in Serbia has utilized the area of valley Valja Fundata for the purpose of flotation tailing dumping since 1961. where 310 000 000 t of tailing is deposited. The valley is situated south of the flotation plant, 1500 m average distance. The most of the Valja Fundata valley terrain is built of the crystal shale with andezite.That type of composition of the terrain (almost 85%) is the cause of its impermeability. On the west side, Valja Fundata valley is naturally partitioned by limestone massif. Before the flotation tailing dump was formed, natural limestone partition had a high graded karsification that was presented by a great number of caverns, cave halls, depressions and funnels, connected with the Valja Fundata cave. The cave entrance is situated on +325 m level and the exit is on +315.5 m level. Down the valley there was a stream of constant flow of Q = 5.0 l/s that has formed the underground flow of 750 m long, through the cave. On its outway, the stream inflows the river Veliki Pek.The Veliki Pek river-bed is tongued into the limestone massif in front of the Valja Fundata cave exit on +295 m level. According the observing of the terrain that was performed before the flotation tailing dump is formed, there were no cracks, neither down the river nor up the river flow, that could have any connection with the Valja Fundata valley creek. Down the river, on 2.5 km distance from the Valja Fundata cave, the rivers Veliki Pek and Mali Pek make the Pek River those inflows in the Danube River.

## 2. Forming of the flotation tailing dump “Valja Fundata”

Before the flotation tailing was deposited into a naturally partitioned area of the Valja Fundata valley, all visible holes and depressions in the limestone massif were fulfilled with crushed stone, open pit waste (andenzite) and concrete. The entrance of the Valja Fundata cave was blocked by open pit waste, and on 457$^{th}$ meter from the entrance it is made a massive concrete barrier. It is the first half of the underground cave area. The second half is facing the exit and it is 293 m long, with the cross cut surface of almost 20 $m^2$. Copper mine Majdanpek deposits the flotation tailing into the “Valja Fundata” dump regarding the principles as the most of the dumps in the world comply. Concerning the environmental protection and the economy effect, in Majdanpek there is an intention to utilize the same flotation tailing dump as long as it's possible; so, after the fulfilling of the deep part of the dump (I phase) there was need to surmount the folds on terrain using the periphery barrages up to +530 m level, made of the hydrocyclonized sand (II phase), while the hydrocyclon overflow went into the dump. In third phase there was a need to surmount of the existing barrages up to +550 m level and for the purpose of the "Valja Fundata" dump rebuilding, the copper mine Majdanpek will apply the new technology of the two stages hydrocyclonized waste – only 3 % of the total waste amount is going to be used on this way but all the rest will be exhausted into the dump. Thereat, the peripheral banks are built in specific way that enabled the great speed of progress in building of the banks and dams and forming of the final external inclines that will be permanently remediated.

### *2.1. Damages at flotation tailing dump "Valja Fundata"*

There are two well-known breaches of the flotation tailing out of this dump. First of them, the smaller one, happened in 1963. and lasted for two hours; it was stopped by natural way – the passage was blocked by deposited flotation tailing. The other one, more serious, happened on January 21, 1974. and lasted until the total amount of water flowed out of the deposition lake, i.e. for 32 hours. Regarding the fact that the flotation tailing was deposited into the naturally partitioned area in the moment of damage, there was no need to cyclonize the tailing. By natural process, the deposited pulp divided in two phases: solid and liquid. After the purging, water had covered the whole area of the deposited tailing and even impacted the ambient terrain. The height of the clear water (deposited lake) above the deposited tailing was 3.35 m in the moment of accident, and the level of the tailing dump mirror was on +454 m. After some time, a connection between the tailing dump area for accumulation and the Valja Fundata cave was created, behind the concrete barrage, caused by the contact of the limestone massif , acidic medium and hydrostatic pressure. The breach of water and flotation tailing appeared on +337 m level, through the abyss of approximately spheral shape of average diameter 1.4-3.0 m (cross cut surface 2.2 $m^2$). When the process of tailings discharge was started, a depression funnel was formed over discharge hole (abyss) (fig.1). Depression was formed on surface of flotation dump and water from deposition lake was uniformly run out into depression funnel where it simultaneously moved (eroded) and held particles of flotation tailings forming new quantity of fluid mass that was participated in general movement. Fluid movement to depression funnel was stable according to monitoring at terrain. When fluid mass reached stream field of depression funnel, stream lines from stable fluid movement were changed into spiral form. Due to size of deposition lake of tailing dump, that is surface and volume of tailing dump at moment of breaking, high heights of discharge column (height of depression funnel) and small abyss diameter, during fluid mass discharging, a contraction (blocking) appeared. Contraction was partial, that was incomplete although people at terrain tried to prevent fluid mass discharge from abyss onto flotation tailing dump bottom, there have not been any results yet, so fluid mass discharge was until complete discharging of water from deposition lake, that was also removed deposited tailings with natural humidity. Total volume of deposited tailings with natural humidity that was discharged during damage was 3.16 millions $m^3$ and 4.5 millions $m^3$ of water what is total of 766 000 $m^3$ of discharged fluid mass into the river Pek.After the flotation tailing flowed out, there left an empty space in the body of the sediment tailing of the dump, in a form of truncated cone.

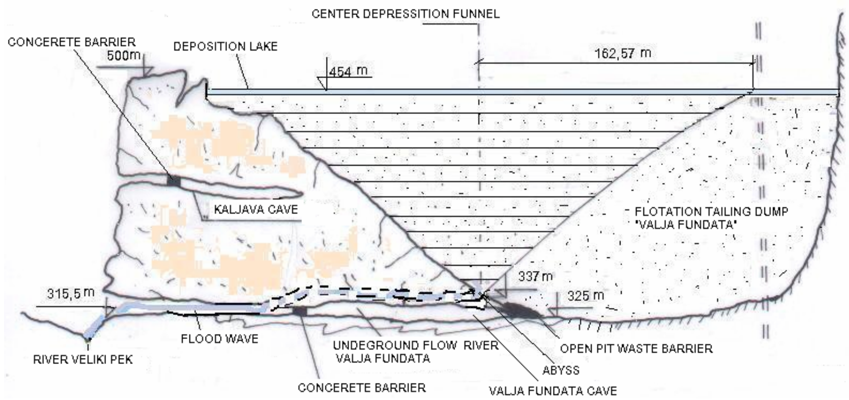


**Fig. 1.** Profile of the natural partition of the limestone massif with caves and the height of the precipitated tailing and clear water in the tailing dump in the moment of accident and the position of the depression funnel

## 3. Modeling of an actual accident happened at the bottom of the flotation tailing through the abyss

For presentation of real damage flow at the flotation tailing dump and possibility of its transformation into similar models and situations in nature, the results were abstracted from concrete ones and presented as ideal. Processing of the existing data of damages on flotation tailing dumps for the aim of obtaining mathematical model, was carried out chronologically as the process of damage was developed in nature.

Mathematical models of damage appeared over abyss on the bottom of flotation tailing dump were formed based on: known volume of discharged tailings with natural humidity (3.16 millions of $m^3$ and change of discharge height at every 10 m per depth of depression funnel (fig.1), water height over deposited tailings in the flotation tailing dump or water quantity in deposition lake, cross cut of damage hole on the tailing dump bottom, rate, quantity and time of fluid mass discharge. Based on known parameters obtained at terrain by monitoring flow of accidents (fig.1), the followings were determined: size of depression funnel, volume of fluid mass discharge, rate and quantity of fluid mass discharge (water and tailings) with change of column height, discharge time, total flood wave in river, pollution concentration in flood wave, range, width and height of flotation material deposit down the river stream and deconcentration of pollution in river (river clearing up). The results are given in tables, graphs and in a form of mathematical model, that is empirical function y = f(x) [1]. Values per X-axis in mathematical model could be changed what could be used in evaluation of the possible damage at any tailing dump with discharge height over abyss lower than 117 m. Average cross cut of damage hole has to be 2.2 $m^2$. Graphical presentation of the results in right-angle coordinate system was carried out based on date from tables and the method of the smallest squares was used for numerical expressions of functions. Determination of functional relationship between two variables was carried out based on real values, obtained by simulation of real damage events on flotation tailing dump in Majdanpek (Serbia). By presentation of the results in coordinate system on x-axis of one variable, and y-axis of the other variable, such curves are obtained. By approximation of this curve, the curve of the following form is obtained:$Y= a\cdot X^b$ ,where: a – coefficient with variable X, b – exponent of variable X. Determination of coefficient and exponent in this curve was carried out by the use of method of the smallest quadrants by the use of computer program EXCEL. By this method use, the mathematical models for determination were obtained [2].

**Mathematical modeling of the discharged tailing volume [3].** The tailing discharge out of the depression funnel, formed during the accident at the bottom of the flotation tailing dump, was recorded while the height of discharge column h (m) decresead – for each part of 10 m. The results of recording are presented by graph on fig. 2: abscissa (x-axes) – the discharge height, ordinate (y-axes) - the volume of the discharged flotation tailing for each of the default levels (segments) of the depression funnel V($m^3$); the intersection of the x and y values gives the points of the curve (fig. 2). Connecting the points we draw the curve. By aproximation of the curve (fig. 2) we get the graphical trend of form V=f(h).

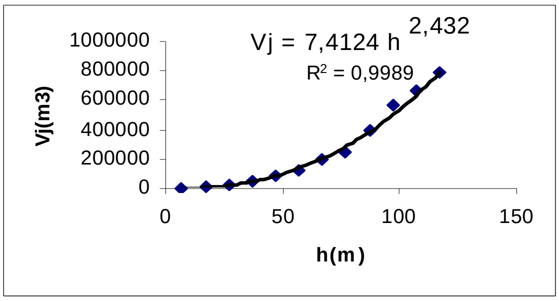


**Fig. 2.** Flotation tailing discharge volume out of the dump depending on the discharge column height

Using the method of the smallest quadrants and the computer program EXCEL, the coefficient and exponent in the curve was determined and also the mathematical model: $V=7.4124h^{2,432}$, where we can change the values of x and: V – known data of the discharged sediment flotation tailing volume, ($m^3$); h –variable of x-axes, refering to the discharge column height, (m). Knowing the function V=f (h) for an actual accident, i.e. flotation tailing breach out of the “Valja Fundata” dump in Majdanpek and using the mathematical model $V=7.4124h^{2,432}$, it is possible to prognose the similar situations, possible discharge tailing volumes for any flotation tailing dump which height of the depositetd tailing withthe deposited lake is not over 117 m. The same principle stands for any of the mathematical models presented in this paperwork.

**Mathematical modeling of the path line of the depression funnel of the flotation tailing discharge [3]** through the abyss at the bottom of the “Valja Fundata” dump – Majdanpek, was based on the known data of the discharged tailing volume in the depression funnel with the change of the discharge column truncated cone. Graphic presentation of the results in orthographic coordinated system (fig. 3) and application of already described process, the mathematical model of the radius of depression funnel path line, in form defined by function: $r=0.64\ V^{0.4003}$, (m), where: r – radius of the depression funnel path line (m); V – variable of X-axes, reffering to discharged volume of the deposited flotation tailing ($m^3$) for each of the observed levels (segments) of the depression funnel.

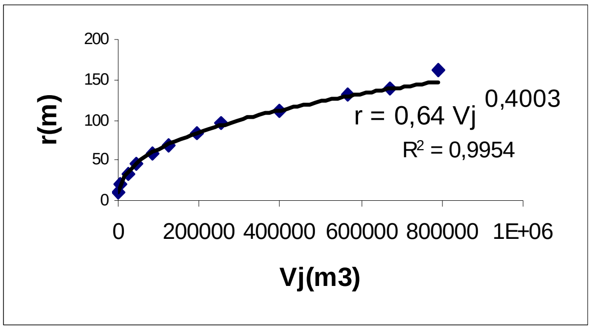


**Fig. 3.** Dependance of the depression funnel path line from the discharged tailing volume for each of the observed segments

**Mathematical modeling of the water inflow [3]** from the deposited lake into the funnel above the abyss on flotation tailing dump “Valja Fundata” – Majdanpek was carried out based on known data of the discharged sediment tailing volume and discharged water quantity for each of the levels of the depression funnel. The addition of the the discharged sediment tailing volumes and the water inflows is, in fact, total volume of the discharged fluid mass with the change of the discharge depression funnel height, fig. 4. Mathematical model has the form: $V_v=10.556\ h^{2.432}$, $m^3$; where: $V_v$ – inflow water volume, ($m^3$), out of the deposited lake into the depression funnel , h – depression funnel height, (m), for each of the observed levels and it is variable of x-axes.

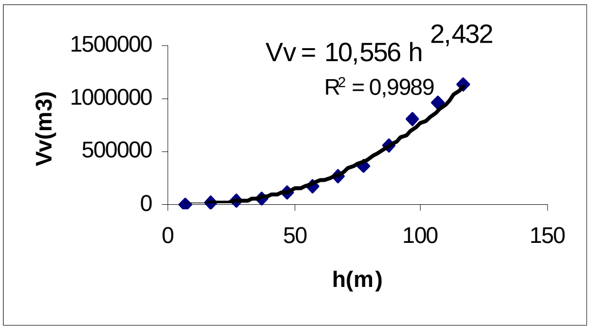


**Fig. 4.** Water inflow from the deposited lake for each of the depression funnel

**Mathematical modeling of the fluid mass discharge through the abyss [3]** at the flotation tailing dump "Valja Fundata" – Majdanpek was carried out based on water inflow from the deposited lake for each of the depression funnel where the water and the sediment tailing formed together new fluid mass in ratio 41.25% of solid and 58.75% of liquid (fig. 5). Mathematical model has the form of function: $V_{f\ m}=17.968\ h^{2.432}\ m^3$, where: $V_{f\ m}$- discharge fluid mass ($m^3$), h – discharge height (m) – variable of x-axes.

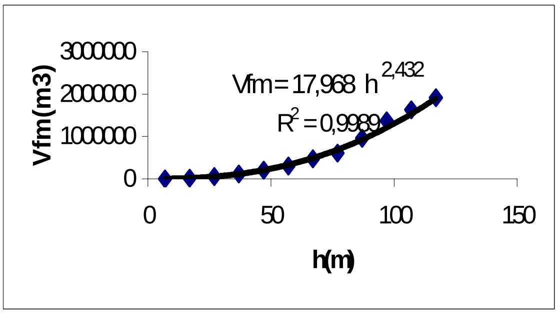


**Fig. 5.** Fluid mass discharge depending on change of the discharge column height

**Discharge speed [3]** of the fluid mass from the abyss at the flotation tailing dump "Valja Fundata"-Majdanpek depending on the discharge column height is presented by graph – fig.6.

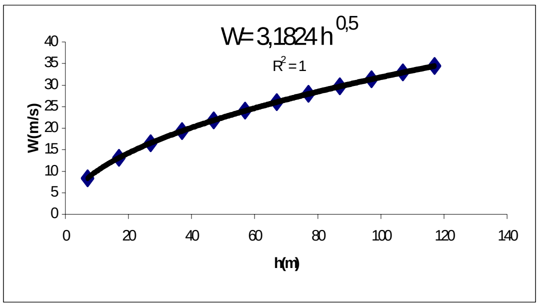


**Fig. 6.** Discharge speed of the fluid mass from the abyss depending on the discharge column height

Mathematical model has a form of function: W=3.1824 h $^{0.5}$ (m/s); where: W – discharge speed of the fluid mass (m/s); h – discharge column height (variable of x-axes) (m).

**Quantities of the fluid mass discharge [3],** Q ($m^3/s$), from the depression funnel are calculated based on the discharge speed for each of the observed levels and average cross cut surface of the abyss. The results are presented by graph – fig.7. The form of the mathematical model is: Q=7.0005 $h^{0.5}$ ($m^3/s$), where: Q-quantity of the fluid mass discharge from the abyss ($m^3/s$), h – depression funnel discharge height (m) and it is the variable of x-axes.

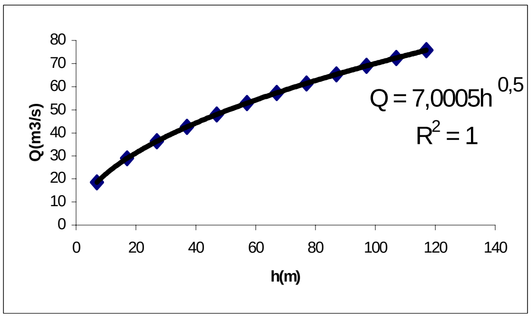


**Fig. 7.** Quantities of the fluid mass discharge from the abyss for each of the observed segments

**Mathematical model of the fluid mass discharge time analysis [3]** at the flotation tailing dump "Valja Fundata" –Majdanpek was determined based on the ratio of the fluid mass volume change for each of the observed levels of the depression funnel and the quantity of discharge. The obtained results are presented by graph – fig.8. Mathematical model of the fluid mass discharge time is: $t=0.2596\ V_{fm}^{\ 0.794}$, where: $V_{fm}$. – fluid mass discharge volume for each of the observed levels ($m^3$); t - time of the fluid mass discharged of the levels (segments).Total effective time of the fluid mass discharge from the tailing dump “Valja Fundata” during the accident at the bottom of the dump is: t=115536.07s, or t=32 hours. Fluid mass discharge time obtained by calculation is equal to an actual fluid mass discharge time on terrain.

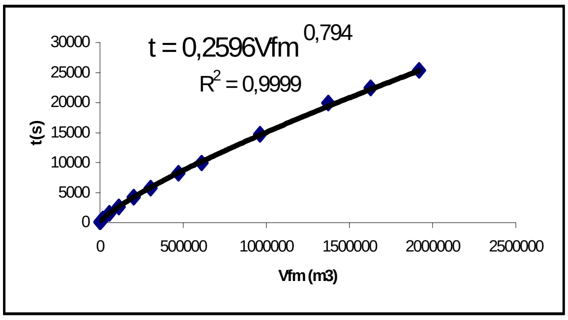


**Fig. 8.** Fluid mass discharge time through the abyss at the bottom of the dump “Valja Fundata”

**Total flow determination [3]** $Q_u$ ($m^3$/s) of the flood wave from the flotation tailing dump “Valja Fundata” – Majdanpek was obtained based on relation: $Q_u=Q+Q_R=55{,}947$ ($m^3$/s),

where:

Q - average quantity of discharged fluid mass from the dump, observed for each of the levels,

$Q_R$ - average river flow ($m^3$/s).

Length, width and height of the flood wave:Range of the flotation tailing particles by the flood wave of the Pek river was determined on terrain (“in situ”) for each of the sections down the river from the flotation tailing dump depending on the flotation tailing precipitated particles diameter (Table 1).

**Table 1.** The size of the flotation tailing sediment particles for each of the flood wave sections

| river valley profile on control points | flotation tailing sediment depending on particles size on control points, $d_{sr}$(m) | length of the flood wave particles transport, L(m) | lateral width of sediment on the control points, $S_n$(m) | thickness of the sediment on the control points, $h_n$(cm) |
|---|---|---|---|---|
| 1-1 | 0.000295 | 2358 | 596 | 85.0 |
| 2-2 | 0.000208 | 4760 | 235.80 | 65.6 |
| 3-3 | 0.000147 | 11230 | 117.30 | 51.3 |
| 4-4 | 0.000074 | 11560 | 98.00 | 37.12 |
| 5-5 | 0.000037 | 26130 | 60.14 | 14.6 |

## 4. Mathematical models of the pollution range, the width and height of the flotation tailing sediment on the flooded surfaces of the river banks

The results of the flood wave heigth investigations on the control points are presented by graph – fig.9, on apscisse (x-axes), l – the width of the leached sides of the river-bed shown on the profiles and it is the variable of x-axes (m), and the ordinate (y-axes) presents $h_{pt}$ – the flood wave heigth of the flow profiles of the Pek river (m). Mathematical model has the form of function:$h_{pt}$=0.0409 l $^{0.7767}$, m [3].

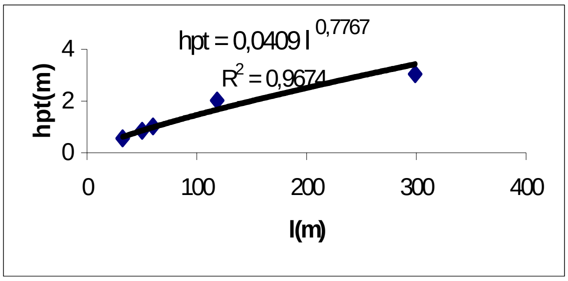


**Fig. 9.** Flood wave height on the control points

Flotation tailing particles range depending on their average diameter, carried by the flood wave to the observed sections of the river flow, is presented by graph – fig. 10. Mathematical model has a form of function: $d_{sr}$=0,2156 L $^{-0.8262}$ (m), where: $d_{sr}$ - average diameter of the particles; L – length of the flood wave particles transport, m [3].

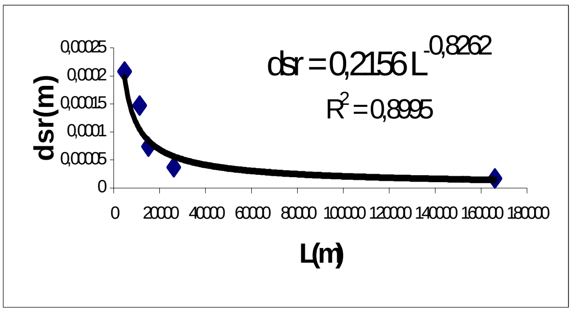


**Fig. 10.** Length of the flotation tailing particles transport depending on the average diameter

The lateral width of the flood wave on the observed flow profiles of the Pek riverwas measured on terrain after the action of the accident, based on the flotation tailing material deposited on left and right side, out of the river-bed. The width of the flood wave depended on the flood wave height and the natural conditions of the terrain (river curves and the river banks inclines). The results obtained by measuring on terrain are presented by graph – fig. 11. Mathematical model of the flood wave width has a form of function: Sn=119.6 $h_{pt}^{\ 1.2826}$ (m), where: Sn – lateral width of sediment, m; $h_{pt}$- height of the flood wave on the flooded areas and it is the variable of x-axes, m [3].

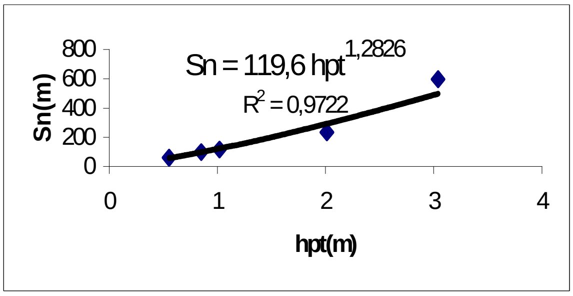


**Fig. 11.** Lateral width of the sediment on the flooded areas of the control points

Average width of the flotation tailing sediment on the observed sections depended on the flood wave height on the flooded areas and average diameter of the flotation material particles on the observed flow profile of the river. The results of measuring on terrain are presented by graph – fig. 12. The form of function of the mathematical model: $h_n$=35.716 $h_{pt}^{0.9128}$ (m) , where: $h_n$ – flotation tailing sediment heigth on the areas out of the river-bed, carried by the flood wave to the observed profile of the Pek river, (m), $h_{pt}$- flood wave height on the flooded areas and it is the variable of x-axes, (m) [3].

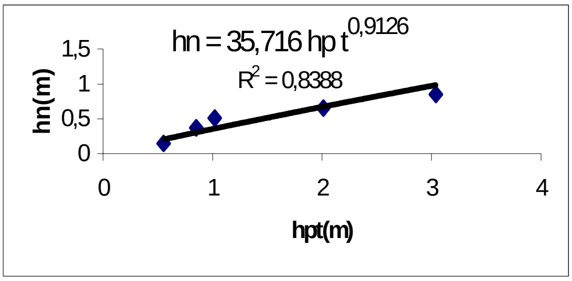


**Fig. 12.** Height of the flood wave on the flooded areas of the control points

The limit between the pulled and suspended sediment in the flood wave on the terrain was proved by the calculation on the length of L = 36.13 km from the accident place. Dilution of the particles in The Pek river appeared when they achieved the size of k = -0.0171359 mm.

## 5. Conclusion

Given mathematical models were applied for the prognosis of the possible accident at the flotation tailing dump “Veliki Krivelj” – Bor, Serbia. The prognosis accident should happen on the collector section at the bottom of the dump, on the connection between the collector (3.0 m diameter) and the outflow element, 1.2 m diameter. On that section of the collector, it was analyzed the stress deformation condition by application of the computer program of the final elements and it was determined that deformation of collector will appear when the height of the tailing sediment reach 50 m.The projected height is 85.22 m. The conclusion was proved on terrain, on the section of the collector under the flotation tailing dump dam, when the heigth of the dam was 56 m and the cracks appeared on the collector; that section of collector is strengthened by inner lamination of 40 cm width, while the profile of the collector is decreased to 2.2 m.

By application of the given mathematical models and change of the values on x-axes, it is possible to evaluate the accidents at the similar flotation tailing dumps if their heights of the deposited tailing are smaller than 117 m, and the average diameter of damage hole at the bottom of the dump is up to 1.4-3.0 m (2.2 $m^2$).

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