

OPTIMUM LOCATING OF ADDITIONAL DRILLHOLES TO OPTIMIZE THE STATISTICAL VALUE OF INFORMATION

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Abstract

In this paper a simulated annealing based algorithm has been presented to locate additional exploratory drillholes based on the data obtained from the previous exploration phases. For this purpose, use has been made of a new criterion, namely the information value resulted from the exploratory drillholes. This criterion is based on the statistical value of information approach and is a logarithmic function of the kriging variance of the estimated block. The required codes for the use of this algorithm have been developed in the Matlab software. Also, a case study has been done in Sungon copper mine for its cross validation.

Key words: additional exploratory drilling; entropy; statistical value of information; optimum locating.

1. Introduction

The number of the additional drillholes drilled to improve the deposit certainty is limited because of the high drilling costs. This limitation has caused extensive study to be carried out in the recent three decades regarding the location of the additional drillholes [1-7] and also evaluation of the data obtained from the additional drilling [8]. The methods presented for the optimum locating of additional drillholes are different combinations of such optimization methods as the mathematical optimization [1], branch and bound [2, 4], simulated annealing [4, 6, 7] and genetic algorithm [5] with geostatistical principles. These methods can be classified in two groups: 1) those that have modeled and solved the problem two dimensionally [1, 2] and 2) those have done it three dimensionally. Only Soltani and Hezarkhan, 2011 [7] have

solved the problem for directional drillholes; the rest [1-6] have assumed all the drillholes to be vertical. In all the previous studies use has been made of minimizing the kriging variance as the objective function and the solution of the problem depends on the characteristics of the deposit spatial structure, the initial drillholes locations and the preliminary assumptions made for the solution. Soltani and Hezarkhani, 2011 [8] proposed some functions for the evaluation of the statistical as well as the real value of information after studying the effects of the addition of the data obtained from drilling. Effort has been made in this paper to present an algorithm for the determination of the optimum location of the additional drillholes using the combination of the statistical value of information function and the simulated annealing method.

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2. Statistical value of information obtained from additional drilling

Information theory is a branch of applied mathematics and electronics engineering that deals with quantifying the information. The history goes back to Claude Shannon's studies about transmission, receiving and optimal storing of data and information [9]. This theory is the basis for the quantitative measuring of information and it also studies different capacities of communication systems and the processed data. Numerous other activities have been done regarding the use of the information theory in such fields as the statistical inference, language processing and coding in different sciences. Information has a special meaning in the "theory of information" which is different from what it means in daily conversations. In the mathematical theory of information, only the surprising aspect of information is considered and its meaningfulness is not given any considerations [10].

According to the theory of information, if X is a random uncertain variable or a vector of such a variable and $f(x)$ is the probability density function, then uncertainty about X may be stated by entropy H as follows [11]:

$$H(X) = - \int f(x) \log_b f(x)$$

In this theory, the basis for the logarithmic function "b" is usually taken equal to 2 [9] and the entropy value is stated in bit units [10].

Suppose variable x in block X_i is not directly measurable, but it may be measured based on the measured variable in the samples taken from drillhole Y . Then, the value of information obtained from drillhole Y in estimating block X_i can be found, based on the mutual information relation, as follows [12]:

$$I(Y, X_i) = H(X_i) - H(X_i | Y)$$

2.1. Value of information before any drilling activities

When there has been no exploratory activity, $H(x)$ and $H(X|Y)$ are equal and $H(X_i) - H(X_i|Y) = 0$; therefore, $I(Y, X) = 0$. Under such conditions, we can determine $H(X)$ or the primary existing entropy and to calculate $H(X)$, we should have the probability function of the resource grade. Since all events between the two boundaries 0 and 100 are of the same probability in this state, it is possible to consider the grade density function as uniform $U(0,100)$ [8]:

$$P(X) = \begin{cases} 0.01 & ; \text{ for } 0 \leq X \leq 100 \\ 0 & ; \text{ for } X \geq 100 \text{ or } X \leq 0 \end{cases}$$

therefore,

$$H(X) = - \sum_0^{100} P(X) \log_2(P(X)) = -100 \times 0.01 \times \log_2(0.01) = 6.64$$

2.2. Value of information after exploratory drilling

Based on the information obtained from drilling, it is possible to provide a block model in which the ore grade distribution has been estimated by such methods as the "nearest neighbor", "inverse square distance weighting" and "kriging". The latter is able to find the estimated grade as well as the estimation variance for every block. This method stands on stationary assumptions and one of the peculiarities of its results is that if the grade probability density function can be modeled as multi-Gaussian, then the error probability density function can be considered as Gaussian with an average equal to zero and a variance equal to an estimation variance of σ_k^2 [13]. Therefore, under such conditions, relation 4 can be developed as follows [8]:

$$\begin{aligned} I(Y, X_i) &= H(X_i) - H(X_i | Y) = H(X_i) - \int_{-\infty}^{+\infty} f(Y) H(X_i | Y = y) dY \\ &= H(X_i) - \int_{-\infty}^{+\infty} f(y) \int f(x|y) \log(f(x|y)) dx dy \end{aligned}$$

Since $f(X|Y)$ is equivalent to the error density function around the estimated point $f(z)$, therefore:

$$I(Y, X_i) = H(X_i) - \int_{-\infty}^{+\infty} f(Y) \int f(z) \log(f(z)) dz dY$$

Supposing that grade follows the normal distribution $N(\mu_y, \sigma_y^2)$, where μ_y and σ_y^2 are the data expectation and variance respectively, since the error follows a normal distribution function $N(0, \sigma_k^2)$, we can write[8]:

$$I(Y, X_i) = H(X_i) - \int_{-\infty}^{+\infty} \frac{1}{\delta_y (2\pi)^{0.5}} \exp\left(-\frac{1}{2} \left[\frac{Y-\mu}{\delta_y}\right]^2\right) \times \int \frac{1}{\delta_k (2\pi)^{0.5}} \exp\left(-\frac{1}{2} \left[\frac{z}{\delta_k}\right]^2\right) \log\left(\frac{1}{\delta_k (2\pi)^{0.5}} \exp\left(-\frac{1}{2} \left[\frac{z}{\delta_k}\right]^2\right)\right) dz dY$$

Using a variable change of $t = \frac{z}{\sqrt{2}\delta_k}$, the above equation will take the following form:

$$I(Y, X_i) = H(X_i) - \frac{1}{\delta_y (2\pi)^{0.5}} \int_{-\infty}^{+\infty} \exp\left(-\frac{1}{2} \left[\frac{Y-\mu}{\delta_y}\right]^2\right) \times \left[-\log(\delta_k (2\pi)^{0.5}) + \frac{\log(e)}{\pi^{0.5}} \int_{-\infty}^{+\infty} e^{-t^2} dt\right] dY$$

Since we have:

$$\int_{-\infty}^{+\infty} e^{-t^2} dt = \frac{1}{2} \pi^{0.5}$$

and

$$\frac{1}{\delta_y (2\pi)^{0.5}} \int_{-\infty}^{+\infty} \exp\left(-\frac{1}{2} \left[\frac{Y-\mu}{\delta_y}\right]^2\right) dY = 1$$

therefore:

$$I(Y, X_i) = H(X_i) - \log(\delta_k (2\pi)^{0.5}) = 6.64 - \log(\delta_k (2\pi)^{0.5})$$

The total value of information gathered from the exploratory drilling is the sum of the values found from estimating all the blocks individually:

$$I(Y) = \sum_{i=1}^N I(Y, X_i)$$

Hence, the value of information obtained from the drilling of the additional drillholes in estimating the block in question, is only a function of the block estimation variance [8].

3. Simulated annealing based algorithm

3.1. Assumptions

The proposed algorithm is based on the following assumptions:

- 1) The deposit is a single element one.
- 2) The deposit is a single population one.
- 3) The variogram model fitted to the data is certain and lacks uncertainty.
- 4) The lengths of all the composites are similar.
- 5) The azimuths and dips of the additional drillholes are specified and they are equal for all of them.

3.2. Explanation

Simulated annealing is a generic probabilistic approach for finding an approximation to the global optimum of a given objective function Φ [13-15]. From a previous solution S_i , another solution S_{i+1} is achieved through a random perturbation of one of the variables in S_i . The acceptance of S_{i+1} as a feasible solution is determined by the Metropolis criterion:

$$P_c(S_i \rightarrow S_{i+1}) = \begin{cases} 1 & \text{if } \Phi(S_{i+1}) \leq \Phi(S_i) \\ \exp\left(\frac{\Phi(S_{i+1}) - \Phi(S_i)}{T}\right) & \text{if } \Phi(S_{i+1}) > \Phi(S_i) \end{cases}$$

where T denotes a positive control parameter (also referred to as the annealing temperature). If S_{i+1} is accepted, a new solution S_{i+2} is derived from S_{i+1} and the probability $P_c(S_{i+1} \rightarrow S_{i+2})$ is calculated with a similar criterion. As the process evolves, the annealing temperature is lowered based on a cooling schedule. This ensures that sub-optimal solutions are accepted with decreasing probability.

The simulated annealing optimization method has been used in such grounds as computer engineering (pinpointing the location of the parts), university curricula, image processing, sampling design and so on,

but one of its applications, which is largely related to the objective of this paper, is its use in the solution of the additional sampling problems [16, 17]. To use this algorithm in finding the location of additional drillholes, it is necessary that first the decision variable, annealing function, objective function and decision mechanism be defined. Figure 1 shows the structure of a simulated annealing algorithm that can also be used in finding the location of the additional drillholes. In this part of the framework and before going into evaluating the cost function, the problem parameters need to be defined. These are the number of additional drillholes that need to be drilled in the mineral resource area. It should be reminded also that for each additional drillholes, the problem variables are two, one for each coordinate. Another important point is the constraints having the aim to keep the optimum locations within the mineral resource region. These are implemented as inequality constraints in the SA, equal to min-max box of the mineral resource region.

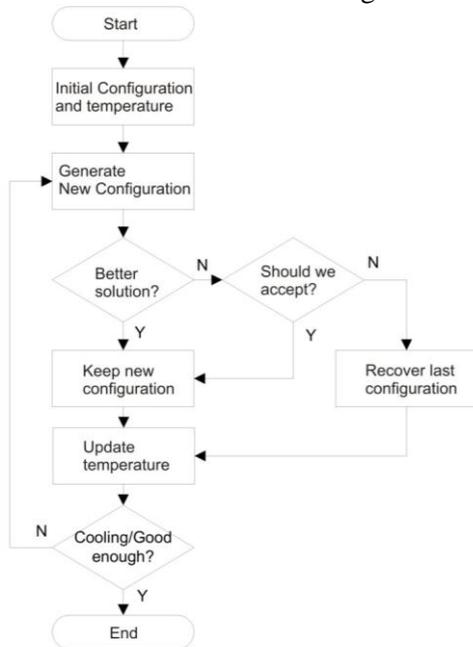


Figure 1. Simulated annealing algorithm

A) Decision variable

The location of drillholes is defined as a permutation of real numbers $P = \{x_1, y_1, \dots, x_j, y_j, \dots, x_n, y_n\}$. There is a real pair (x_j, y_j) in the permutation for every drillhole that shows the collar longitude and latitude in the mineral region. On this basis, the number of the optimization variables is twice that of the additional drillholes.

B) Annealing function

This function is used to create random changes in the combination. The common annealing functions are “Boltzman”’s and “fast”.

C) Objective function

In every step of the simulated annealing algorithm, the new solution is evaluated with respect to the existing one according to the objective function. Based on equations 10 and 11, the statistical value of information, obtained from the additional drilling, can be found from the following equation:

$$I(Y) = \sum_{i=1}^N I(Y, X_i) = \sum_{i=1}^N [H(X_i) - \log(\delta_x(i)(2\pi\epsilon)^{0.5})]$$

where N is the number of blocks and σ_i is the kriging standard deviation of the i^{th} block after adding the data obtained from the additional drillholes to the set of the existing data which is a function of the location of the primary and additional drillholes and the fitted variogram model. Knowing that $H(x_i)$ is not a function of the decision variable and it is a constant number (6.64), we may define the optimization problem of the statistical value of information equivalent to the following constrained minimization problem:

$$\text{Minimize } \sum_{i=1}^N \log(\delta_i(2\pi\epsilon)^{0.5})$$

$$\text{subject to: } x(i) \leq UX ; i = 1, \dots, n$$

$$x(i) \geq LX ; i = 1, \dots, n$$

$$y(i) \leq UY ; i = 1, \dots, n$$

$$y(i) \geq LY ; i = 1, \dots, n$$

where $x(i)$ and $y(i)$ are the longitude and latitude of the i^{th} drillhole respectively, n is the number of the additional drillholes, LX and UX are the longitudes of the western and eastern boundaries of the deposit respectively, and LY and UY are the latitudes of the southern and northern boundaries of the deposit respectively.

To find the objective function, it is necessary that, first the location of the extracted samples be specified and then the estimation variance be calculated. Since it is not possible to definitely determine the exact location of where the drillhole meets the mineral, use was made of the 3D block model

of the deposit and overburden to determine the appropriate location of the extractable samples from each drillhole. To do this, a program was written in the Matlab software so that, based on the location of the drillholes' collars (the combination created in each iteration of simulated annealing) and the 3D block model, it could, first, estimate those parts of the drillholes that cut the mineral and, then, based on the constant sample length (equal to the composite length), determine the location of the samples extractable from each drillhole based on the location of its collar. The pseudo-code related to these calculations is shown in Figure 2.

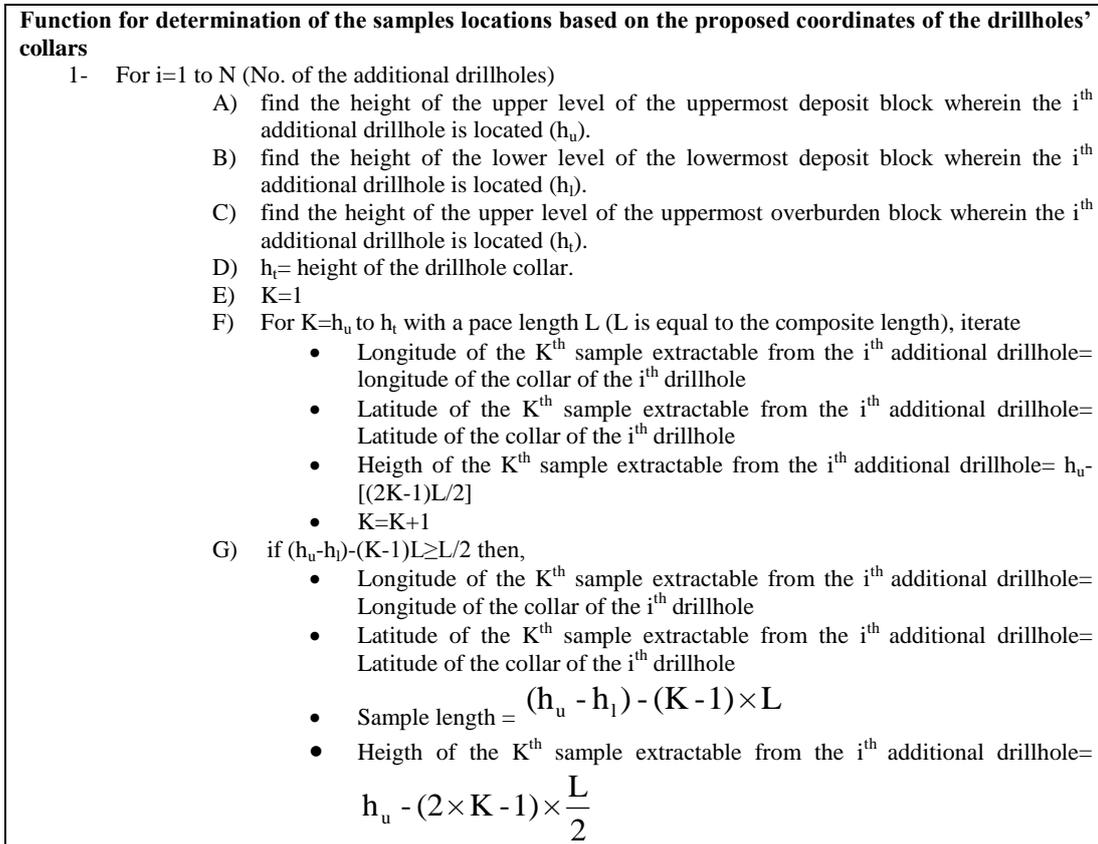


Figure 2. Pseudo-code related to the determination of the location of the samples extractable from the proposed additional drillholes in each iteration of the simulated annealing

D) Acceptance criteria

To find the acceptance or refusal probability of the new combination when $\Delta\text{Energy} > 0$, use was made of Boltzman function [20].

4. Evaluation of the algorithm proposed for Sungon copper mine

The proposed algorithm should be evaluated from two aspects: 1) the parameters used in the simulated annealing, and 2) the performance.

4.1. Data

To evaluate the proposed algorithm, Sungon copper mine in East Azarbaijan province, Iran, was selected. The mine contains 500 million tons of copper sulfide with an average grade of 0.76 % copper and 0.01% molybdenum. Exploratory studies have revealed that Sungon mineral body is elliptical in shape down east-westerly with a major axis of approximately 2.2 kilometers and a minor axis of nearly 1.1 kilometer [18]. The database of Sungon drillholes contains the data of 148 exploratory drillholes out of which 98 are primary and 50 are additional. Figure 3 shows the dispersion pattern of all the drillholes in the area. Since the locations of the additional drillholes in this area have been specified, it is possible that a general evaluation of the proposed algorithm be carried out based on a comparison between the existing results and those obtained latter if the related studies are ever done.

As shown in Figure 3, most of the additional drillholes are located in the southern part of the deposit and only a few are scatteredly drilled in the northern part. This is the reason why use has been made of the data of the southern part containing 32 primary drillholes.

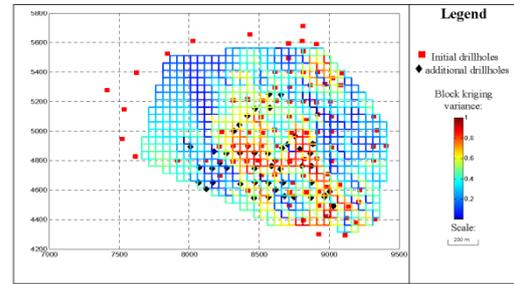


Figure 3. Dispersion pattern of the primary and additional drillholes dug in Sungon area and the block model prepared for Sungon copper deposit at 1735 m level (red squares show the locations of the primary drillholes and black lozenges show those of the additional ones)

The geological model of the deposit's southern part was prepared based on the finite modeling method and converted into the block model based on a $50 \times 50 \times 50$ block dimensions. Also, the 3D experimental variogram of the copper grade (figure 4) was calculated and a double-structure spherical model with the parameters $C_0=0$, $C_1=0.07$, $C_2=0.05$, $a_1=71.5$ and $a_2=208$ was fitted to it.

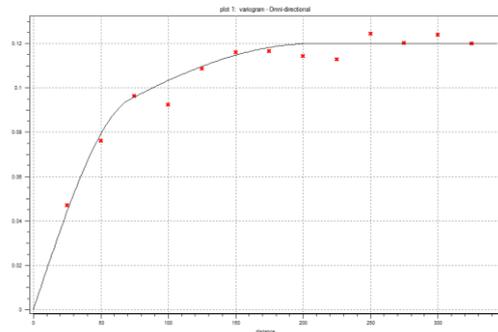


Figure 4. Experimental variogram of copper grade in the primary drillholes and the model fitted to it

4.2. Studying the convergence behavior of the proposed algorithm

In most cases, what is done with the purpose of optimization is, in fact, an improvement. Optimization is to reach the optimal point and it comes after improvement. This definition has two parts:

- 1) determination of the optimum point, and
- 2) improving the process of this determination [19].

Figure 5 shows the convergence trend during the implementation of the additional drillholes locating algorithm for the data of Sungun southern part.

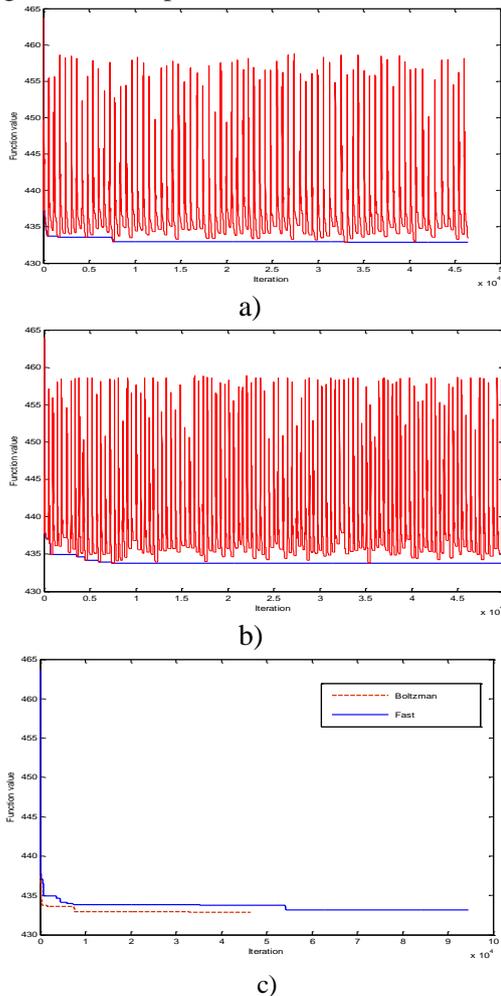


Figure 5. Convergence trend during the implementation of the additional drillholes locating algorithm based on the statistical value of information approach in different iterations. a) Boltzman annealing function and the initial temperature of 465, b) fast annealing function and the initial temperature of 465, c) comparison of the performances of the two annealing trends

The initial temperature was found based on the Kirckpatrick (13) algorithm (464°C) and the number of iterations before reannealing was 20 times the number of optimization variables, i.e. 800. Also, the number of reannealing was taken to be between 10 and 50 [20]. Figures 5(a) and 5(b) show the max. and the min. values of the objective function accepted throughout the optimization process based on different annealing functions (ie. Boltzman and fast annealing functions). It is to be noted that in the initial phase (when the temperature is high), the acceptance probability of the combination of which the objective function value has less desirability with respect to the existing one, is more and it decreases with a fall in the temperature. After every reannealing, when there is a sudden rise in the temperature, again the combinations with more energy have been accepted. Figure 5(c) shows that Boltzman annealing function's performance is better than that of the fast one in optimum locating of the additional drillholes. As shown in figure 6, the temperature has risen again during the reannealing process after 800 iterations and hill climbing has occurred.

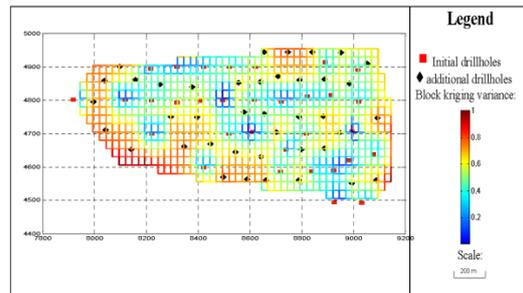


Figure 6. Locations of the additional drillholes proposed by the algorithm based on the statistical value of information approach (block color is based on the estimation variance)

4.3. Results evaluation

The proposed optimum locations for the additional drillholes are shown in figure 6. Figure 7(a) shows the value of information

from the primary drillholes, figure 7(b) shows the value of information from the exploratory drillholes after adding the data from the additional drillholes at the dug points, and figure 7(c) shows the value of information after adding the data from the additional drillholes at the proposed points. A comparison of these three figures shows that drilling the additional drillholes at the proposed points has not only caused the highest increase in the statistical value of information in all the blocks, but the value has also increased more homogeneously in all the blocks.

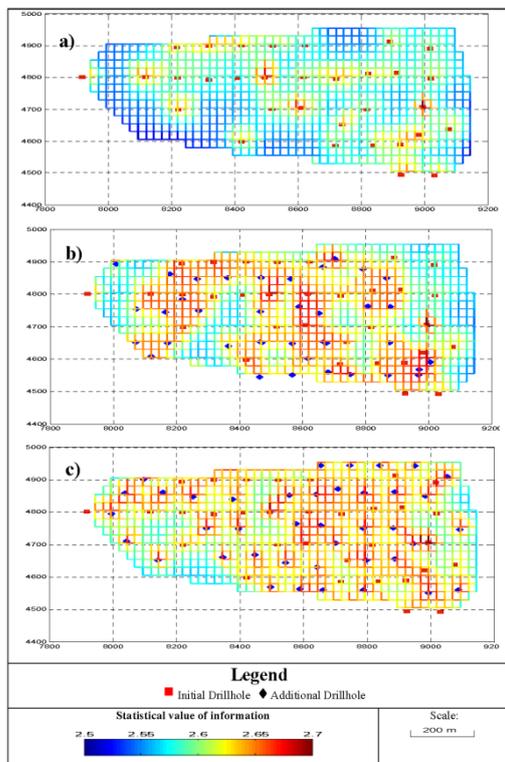


Figure 7. Maps of the value of information for every block obtained from digging. a) based only on the information of the primary drillholes, b) based on the information of the primary drillholes and that of the additional ones dug in the area, and c) based on the information of the primary drillholes and that of the additional ones proposed by the algorithm

5. Conclusions

All the previous studies on the issue of optimum locating of additional drillholes are based on minimizing the kriging variance. Efforts has been made in this paper, for the first time, to make use of the statistical value of information as a new criterion for this purpose. Selection of the block dimensions is a factor that affects the precision of the studies. Reducing block dimensions will increase not only the precision of assessing the samples extractable form the additional drillholes, but also the preciseness of determining the objective function. Yet, this reduction will increase the time needed for calculations for two reasons: 1) the number of blocks will increase which causes an increase in the number of points for which the objective function has to be determined as a function of the estimation variance, and 2) the composite length should also reduce which means an increase in the number of samples, a bigger kriging matrix, and more time to solve it for every block. Although the function of the statistical value of information makes possible the comparison of two different information sets, it lacks applicability in evaluating economic information and it is necessary that the additional drilling information be evaluated from the economic point of view in later studies. This research may be continued, using a combination of heuristics and direct search methods, to reduce the calculation time.

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