

APPLICATION OF ASSIGNMENT PROBLEM IN PROJECT REALISATION

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Abstract

The assignment problem belongs to the group of optimization problems. This paper gives an example of using assignment problem solving for a process optimization in mining and mineral processing.

Key words: assignment, optimization, hungarian algorithm.

1. Introduction

One of the very important goals of project realization is its realization within the limits of the planned costs. When dealing with large projects, the right assignment of the workforce and of other resources can have a significant influence on the project total costs.

2. The model of assignment

The main purpose of this model is the assignment of a certain number of workers or resources intended for specific jobs.

The optimization of the assignment of workers or resources in charge of specific jobs is realized through the model in

which the function of the goal represents the demand for the minimization of costs of the given assignment, and can be represented as follows:

$$(\min) Z = \sum_{i=1}^m \sum_{j=1}^n c_{ij} x_{ij} \quad (1)$$

Whereby x_{ij} stands for the variable which shows engagement or nonengagement of worker i in charge of job j , and c_{ij} stands for the costs worker i makes during job j .

If worker m is supposed to be assigned job n , whereby the assignment is such that it requires minimal costs for doing the job, the assignment model will be the following:

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$$\begin{aligned}
 (\min) \quad Z &= \sum_{i=1}^m \sum_{j=1}^n c_{ij} x_{ij} \\
 \sum_{i=1}^m x_{ij} &= 1 \quad i = 1, 2, \dots, n \\
 \sum_{j=1}^n x_{ij} &= 1 \quad j = 1, 2, \dots, n
 \end{aligned} \tag{2}$$

The variable x_{ij} can be either 0 or 1, and it shows engagement, that is non-engagement, of worker i for job j . Thus defined model represents a special form of the linear programming model (the so-called 0-1 programming).

In order to solve the assignment problem, the number of workers (or other resources) has to be equal to the number of jobs ($m=n$).

The importance of solving the problem defined in this way lies in the fact that there are a large number of possible assignments, but optimization means that

a worker is assigned a job which secures the lowest total costs.

3. Mathematical statement of assignment problem

Given the $n \times n$ matrix (C_{ij}) of real numbers, find a permutation p ($p_i, i=1, \dots, n$) of the integers $1, 2, \dots, n$ that minimizes

$$(\min) \sum_{i=1}^n c_{ip_i} \tag{3}$$

For example the 4 X 4 matrix

$$C = \begin{bmatrix} 22 & 12 & 16 & 15 \\ 20 & 12 & 24 & 20 \\ 30 & 16 & 22 & 30 \\ 8 & 12 & 10 & 8 \end{bmatrix}$$

there are 24 possible permutations ($4! = 4 \times 3 \times 2 \times 1 = 24$).

The possible permutations and the associated sums are given in table 1.

Table 1. The possible permutations and the associated sums

	P	$\sum_{i=1}^n a_{ip_i}$
(1)	1 2 3 4	64
(2)	1 2 4 3	74
(3)	1 3 2 4	70
...		
(22)	4 2 3 1	57
...		
(24)	4 3 2 1	63

Permutation 22 gives the smallest sum, that is, permutation 4 2 3 1 and the

associated sum $\sum c_{14} + c_{22} + c_{33} + c_{41} = 15+12+22+8$ which is 57.

4. An algorithm for the assignment problem

The so-called Hungarian Algorithm, named after Hungarian mathematician D.Konig who devised it, is used for solving model (2). This numerical method is based on minimizing the so-called opportunistic costs that are the result of non-engagement of the most efficient worker.

The method of assignment optimization is based on n usage of matrix C whose elements represent the goal function, that is, indicators of workers' (resources) efficiency in doing the jobs (activity).

If m workers are needed for doing n jobs, efficiency matrix C is as follows:

$$C = \begin{bmatrix} c_{11} & c_{12} & \dots & c_{1n} \\ c_{21} & c_{22} & \dots & c_{2n} \\ \dots & \dots & \dots & \dots \\ c_{n1} & c_{n2} & \dots & c_{nm} \end{bmatrix} \quad (4)$$

Whereby c_{ij} stands for the costs of i -resource while doing activity j .

Finding the optimal assignment within which the right resources are found for a specific activity ensuring the minimal costs is realized in three main phases (see Figure 1):

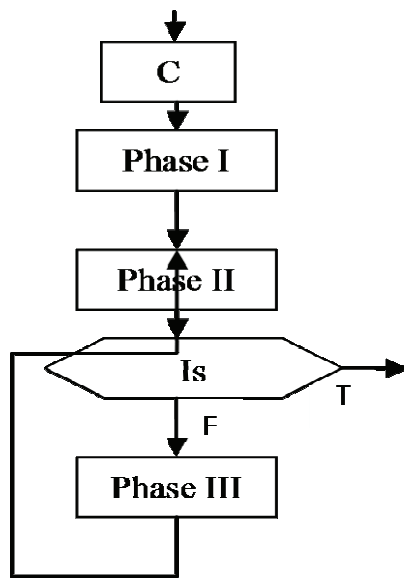


Fig. 1. Selection of optimal disposition (Munkers' Algorithm)

a) Optimization Phase I

For each row of the matrix, find the smallest element and subtract it from every element in its row.

After that the similar approach is applied to the rows of the matrix, that is, for each row of the matrix the smallest element is found, which is then subtracted from every element in its row of the matrix.

The application of these steps in Phase I leads to forming matrix C' which has in each row at least one element c_{ij} whose value is 0.

Matrix C' is the starting matrix from which we continue the optimization process in phases II and III.

b) Optimization Phase II

In matrix C' we identify the so-called starred zeros and primed zeros.

The simplest way of identifying starred and primed zeros is the following:
 - The zeros within the rows are starred while the remaining zeros within the corresponding columns are primed.

- In the rows with a greater number of zeros, the starred zeros are assigned to the remaining rows starting from the rows with the smallest number of zeros paying attention to the fact that there can be only one starred zero in each row.

The optimal solution is found if the number of starred zeros is equal to the number of rows of the matrix, whereby the starred zeros identify the elements in the matrix whose sum represents the optimal assignment problem solving.

Otherwise we use vertical and horizontal lines to 'cover' the rows and columns in matrix C' which contain a zero value, whereby the minimal number of lines is used to cover the rows and columns with a zero value, and then we proceed to the third optimization phase.

b) Optimization Phase III

This phase of the process of finding the optimal solution is realized following the given steps:

- defining the minimal element in matrix C' which is not covered,
 - the minimal noncovered element in matrix C' is subtracted from all noncovered elements in the matrix,

- the minimal noncovered element in matrix C' is added to the twice covered elements in the matrix,

- the remaining, once covered elements in matrix C' remain unchanged.

After the third phase, matrix C' is modified and the same process is repeated in phases II and III until the number of primed zeros is equal to the number of rows in matrix C' .

5. Numerical examples

We will show the assignment problem solving through some examples given in the book *Scientific Bases of Project Management (Naučne osnove upravljanja projektima)*:

Example 1:

There are four trucks (A, B, C, D) and four truck drivers for the transport of a larger amount of material (broken stone) needed for the construction of a part of the road. It has been measured and determined that each driver needs a specific period of time for each round, as is shown in table 2.

Table 2. Specific period of time in the function of trucks and trucks driver

Truck Driver	A	B	C	D
I	22	12	16	15
II	20	12	24	20
III	30	16	22	30
IV	8	12	10	8

That is,

$$C = \begin{bmatrix} 22 & 12 & 16 & 15 \\ 20 & 12 & 24 & 20 \\ 30 & 16 & 22 & 30 \\ 8 & 12 & 10 & 8 \end{bmatrix}$$

After the first optimization phase, we have the following matrix C' :

$$C' = \begin{bmatrix} 10 & 0^* & 2 & 3 \\ 8 & 0 & 10 & 8 \\ 14 & 0 & 4 & 14 \\ 0 & 4 & 0^* & 0 \end{bmatrix}$$

Since the number of starred zeros (0^*) is smaller than the number of the rows in matrix C' after the first optimization phase, the optimization process is continued through the second and then the third phase.

The optimal solution is found after the third iteration after which matrix C' is as follows:

$$C' = \begin{bmatrix} 7 & 2 & 0 & 0^* \\ 3 & 0^* & 6 & 3 \\ 9 & 0 & 0^* & 9 \\ 0^* & 9 & 1 & 0 \end{bmatrix}$$

The optimal assignment of the drivers and the trucks and the needed working hours for the job is shown in table 3.

Table 3. The optimal assignment of the drivers and the trucks

Driver	Truck	Time (min)
1	D	15
2	B	12
3	C	22
4	A	8
Total time:		57

No other assignment gives shorter time than this one (57 minutes).

Example 2:

There are four workers in charge of flotation at disposal for the four technologically separated lines of flotation (A, B, C, D). The process of measuring the technological results which the workers accomplish at certain sections of

flotation shows that the workers in charge of flotation at certain sections cause different losses of metal in the barren ground. Table 4 shows the results of monitoring losses in metal in the barren ground.

The workers in charge of flotation should be arranged along the sections of flotation in such a way to ensure the minimal losses of metal in the barren ground.

Table 4. The results of monitoring losses in metal in the barren ground

Worker	Flotation section	Losses of metal in the barren ground (%)			
		A	B	C	D
1		10	8	9	12
2		15	12	14	10
3		12	15	17	12
4		9	11	13	8

That is,

$$C = \begin{bmatrix} 10 & 8 & 9 & 12 \\ 15 & 12 & 14 & 10 \\ 12 & 15 & 17 & 12 \\ 9 & 11 & 13 & 8 \end{bmatrix}$$

After the first optimization phase, we have the following matrix C' :

$$C' = \begin{bmatrix} 2 & 0^* & 0 & 4 \\ 5 & 2 & 3 & 0^* \\ 0^* & 3 & 4 & 0 \\ 1 & 3 & 4 & 0 \end{bmatrix}$$

Since the number of starred zeros (0^*) is smaller than the number of the rows in

matrix C' after the first optimization phase, the optimization process is continued through the second and then the third phase.

The optimal solution is found after the third iteration after which matrix C' is as follows:

$$C' = \begin{bmatrix} 3 & 1 & 0^* & 7 \\ 4 & 0^* & 0 & 0 \\ 0^* & 2 & 3 & 1 \\ 0 & 1 & 1 & 0^* \end{bmatrix}$$

Table 5 shows the optimal arrangement of the workers in charge of flotation which allows the minimal losses of metal in the barren ground.

Table 5. The optimal arrangement of the workers in charge of flotation

Worker	Flotation line	Losses of metal in the barren ground (%)
1	C	9
2	B	12
3	A	12
4	D	8
Average loss of metal:		10.25

No other assignment gives shorter average losses of metal in the barren ground than this one (10.25 %).

6. Conclusion

Optimization of the assignment of workers (and other resources) for doing particular jobs (or activity) leads to a significant reduction of the costs, which is especially important when planning large projects.

The larger the number of alternatives (n), the larger the number of the possible assignment varieties ($n!$ varieties). Nowadays computers can solve this problem of varieties and they can do that rather fast. Yet, the Hungarian or Munkres' Assignment Algorithm is still

an effective solution to this problem, even when we use a computer.

There are numerous varieties (modifications) of the above mentioned algorithm. However, we used our own version devised at the Faculty of Management in Zajecar for the example given in this thesis.

7. References

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